MTH 203: Groups and Symmetry

Homework V

Problems for practice

1. Establish the two assertions 1.4 (ii) (a) of the Lesson Plan.
2. Establish the assertions in 3.1 (vii) of the Lesson Plan.
3. Establish the assertion in 3.2 (i) of the Lesson Plan in full detail.
4. Consider the maps described in 3.3 (ii) of the lesson plan.
   (a) Show that these are homomorphisms.
   (b) Find the kernel and image of each of these maps.
   (c) Determine which among these maps are epimorphisms, or monomorphisms, or isomorphisms.
   (d) Use the First isomorphism theorem to draw suitable conclusions for each of these maps.
5. For each of the groups $\mathbb{Z}_8$, Q_8, and D_8, determine the following. Give rigorous arguments to justify your answers.
   (a) The number of subgroups of orders 2 and 4.
   (b) The normalizers and centralizers for each of the subgroups listed in (a).
   (c) Which among the order 4 subgroups enlisted in (a) are normal, cyclic, or non-cyclic (i.e having the same structure as U_8).
   (d) The center of the group.

   Use this information to find all aspects that make these groups structurally distinct (or non-isomorphic) groups.
6. Classify all homomorphisms from:
   (a) $\mathbb{Z} \to \mathbb{Z}$.
   (b) $\mathbb{Z}_m \to \mathbb{Z}_n$
   (c) $\mathbb{Z}_m \to \mathbb{Z}$
   (d) $\mathbb{Z} \to \mathbb{Z}_n$